Transformation of the orbital angular momentum at the reflection and transmission of a light beam on a plane interface

2008 J. Phys. A: Math. Theor. 41 505202

(http://iopscience.iop.org/1751-8121/41/50/505202)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 505202 (19pp) doi:10.1088/1751-8113/41/50/505202

Transformation of the orbital angular momentum at the reflection and transmission of a light beam on a plane interface

V G Fedoseyev

Institute of Physics, University of Tartu, Riia 142, 51014 Tartu, Estonia

E-mail: fedo@fi.tartu.ee

Received 9 June 2008, in final form 14 October 2008
Published 10 November 2008
Online at stacks.iop.org/JPhysA/41/505202

Abstract
The transformation of the orbital angular momentum (OAM) at the reflection and transmission of a paraxial light beam on a plane interface of two isotropic transparent media is considered. The unitary formula for the intrinsic OAMs of the incident, reflected and transmitted beams is derived. The processes of the change of the intrinsic OAM at reflection and transmission, and the appearance of the extrinsic OAMs, which are generated by the respective shifts of the centres of gravity of secondary beams, are analysed on the basis of the conservation laws.

PACS numbers: 42.25.−p, 42.25.Gy, 42.60.Jf

1. Introduction

The light beams carrying the intrinsic orbital angular momenta (IOAM) have been of great interest during the last 16 years. Numerous works have been devoted to the investigations of the features of such beams and of their interaction with the matter (see, for instance, [1–3] and the references in [2, 3]).

In this paper, we will consider the transformation of the OAM at the reflection and transmission of the light beam on a plane interface of two isotropic transparent media. The united formula for the IOAMs per unit length (pul) of the incident, reflected and transmitted beams will be derived.

The obtained results will be analysed from the point of view of the conservation laws, which take place in the process of reflection and transmission of light packets. It has been shown in [4] (see also [5–7]) that the following quantities are the invariants of this process: the total electromagnetic energy, the component of the total linear momentum (LM) parallel to the interface, and the component of the total AM normal to the interface. It should be mentioned that the second and the third conservation laws do not follow directly from the symmetry properties of the system under consideration, because the electromagnetic field in a matter is not a conservative system. These laws are valid, provided that the LM and the AM are defined in the sense of Minkowski; further on such a definition will be assumed in this paper. They are invalid for the above-mentioned components of the LM or the AM defined in the sense of Abraham, although the LM and the AM so defined are recognized to be 'true' (see, for instance, [8, 9]). The conservation of the normal component of the total AM entails the following consequence: if the normal component of total IAM changes after reflection and transmission, the centres of gravity of the secondary packets must suffer the shifts in the transverse direction, i.e. perpendicular to the plane of incidence, in order that the normal component of the total extrinsic AM generated by the shifts could compensate this change. The calculations of the IOAM-dependent transverse shifts of the centres of gravity (TSCG) of the partially reflected and transmitted beams have been performed in [10–12]; the experimental detection of the TSCG of the former has been made by Dasgupta and Gupta [13]. In the present paper, the problem of the relation of the conservation laws to the IOAM-dependent TSCGs will be analysed.

A similar problem, i.e. the relation of the conservation laws to the spin-dependent TSCGs, has been under consideration for a long time. It has been recognized long ago that there is a single-valued relation between the spin-dependent transverse shift of the totally reflected beam, which is usually called the Imbert–Fedorov shift [14, 15], and the change of the normal component of the spin AM at total reflection; for this reason, de Beauregard has called this shift 'the translational inertial spin effect with photons' [16]. Interest in this problem has been resumed in the last few years [17–22]. It was initiated by the letter by Onoda *et al* [17], who have drawn attention to the fact that in some cases, in particular in the important case when the incident beam is circularly polarized and its polarization does not depend on the transverse coordinate, the spin-dependent TSCG of every secondary beam looks as if the conservation law of the normal component of the total AM takes place for the processes of partial reflection and transmission separately.

In this paper, the processes of transformation of the IOAM and the spin IAM at reflection and transmission will be compared.

2. Geometry of reflection and transmission

We shall consider the reflection and transmission of a monochromatic light beam at a plane interface of two semi-infinite transparent, isotropic, nondispersive and nonmagnetic media. The scheme of this process is shown in figure 1. The position of the interface is defined by the equation $\hat{\mathbf{N}} \cdot \mathbf{r} = 0$, where $\mathbf{r}$ is the 3D radius vector, and $\hat{\mathbf{N}}$ is the unit normal to the interface directed from the first medium, which fills the upper half-space (see figure 1), to the second one. The beam is assumed to be incident from the first medium. Let us denote the refractive indices of the media in the upper half-space ($\hat{\mathbf{N}} \cdot \mathbf{r} < 0$) and the lower half-space ($\hat{\mathbf{N}} \cdot \mathbf{r} > 0$) by $n^{(1)}$ and $n^{(2)}$, respectively.

Throughout this paper, we shall use the superscripts i, ρ, and τ for indicating the quantities characteristic of the incident, reflected and transmitted beams, respectively; the superscript a will be used in order to designate an arbitrary (incident, reflected or transmitted) beam, so, $a = i, \rho$ or τ. We shall employ three Cartesian coordinate systems connected with the incident, reflected and transmitted beams; these systems will be termed the i-, ρ- and τ-systems.

The $z^{(i)}$-axis is assumed to coincide with the incident beam's axis (the exact definition of the latter will be given later). This axis and the unit vector $\hat{\mathbf{N}}$ define the position of the beam's plane of incidence in 3D space; the angle between the unit vectors $\hat{\mathbf{N}}$ and $\hat{\mathbf{z}}^{(i)}$ is the beam's angle of incidence: $\theta^{(i)} = \arccos(\hat{\mathbf{N}} \cdot \hat{\mathbf{z}}^{(i)})$. The coordinate origin O in every system is

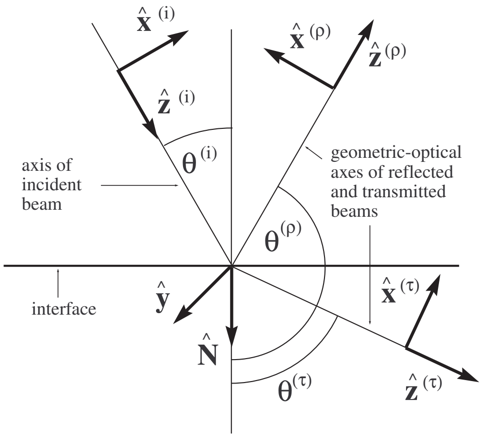

Figure 1. Geometry of reflection and transmission. Orientation of the $z^{(\tau)}$-axis relative to the $z^{(i)}$-axis corresponds to the case when $n^{(2)} < n^{(1)}$, and $\sin\theta^{(i)} < n^{(2)}/n^{(1)}$.

taken to be the point of intersection of the incident beam axis with the interface. In the ρ- and τ-systems, the $z^{(\rho)}$ and $z^{(\tau)}$ axes are assumed to coincide with the geometric-optical axes of the reflected and transmitted beams. The geometric-optical axes of the secondary beams are defined as the rays which are intersected by the interface at the coordinate origin and whose directions are characterized by the unit vectors $\hat{\mathbf{z}}^{(\rho)}$ and $\hat{\mathbf{z}}^{(\tau)}$, these vectors being related to $\hat{\mathbf{z}}^{(i)}$ through the Snell laws. The beam's angles of reflection and transmission $\theta^{(\rho)}$ and $\theta^{(\tau)}$ are the angles between the vector $\hat{\mathbf{N}}$ and the respective geometric-optical axis. $\theta^{(\rho)} = \pi - \theta^{(i)}$, while $\theta^{(\tau)}$ is defined by the relation $n^{(2)}\sin\theta^{(\tau)} = n^{(1)}\sin\theta^{(i)}$. Let us denote by $\nu^{(a)}$ the relative refractive index of the medium, in which the ath beam propagates, i.e. $\nu^{(i)} = \nu^{(\rho)} = 1$, and $\nu^{(\tau)} = n^{(2)}/n^{(1)}$. For an arbitrary beam, the parameter $\nu^{(a)}$ can be written as

$$\nu^{(a)} = \sin\theta^{(i)}/\sin\theta^{(a)}. \tag{1}$$

We shall also use the other parameter of the ath beam

$$\zeta^{(a)} = \cos\theta^{(i)}/\cos\theta^{(a)}. \tag{2}$$

It is evident that $\zeta^{(i)} = -\zeta^{(\rho)} = 1$.

Let us define the y-axis of every system to be perpendicular to the plane of incidence; this axis (the transverse one) is characterized by the unit vector $\hat{\mathbf{y}} = \hat{\mathbf{N}} \times \hat{\mathbf{z}}^{(i)}/|\hat{\mathbf{N}} \times \hat{\mathbf{z}}^{(i)}|$. The $x^{(i)}$, $x^{(\rho)}$ and $x^{(\tau)}$ axes lie in the plane of incidence; the direction of the $x^{(a)}$-axis is given by the unit vector $\hat{\mathbf{x}}^{(a)} = \hat{\mathbf{y}} \times \hat{\mathbf{z}}^{(a)}$.

In the ath system, the 3D radius vector is represented as follows:

$$\mathbf{r} = \mathbf{r}_\perp^{(a)} + z^{(a)}\hat{\mathbf{z}}^{(a)}, \tag{3}$$

where $\mathbf{r}_\perp^{(i)}$, $\mathbf{r}_\perp^{(\rho)}$ and $\mathbf{r}_\perp^{(\tau)}$ are the 2D planar radius vectors lying in the planes which are perpendicular to the axis of the incident beam or to the geometric-optical axis of the respective secondary beam:

$$\mathbf{r}_\perp^{(a)} = x^{(a)}\hat{\mathbf{x}}^{(a)} + y\hat{\mathbf{y}}. \tag{4}$$

3. Intrinsic and extrinsic OAMs of the incident, reflected and transmitted beams

3.1. The electromagnetic field of the ath beam

Let us assume that the incident beam is paraxial, and that it is mainly *s*- or *p*-polarized. First assumption means that the relation

$$\lambda/b \ll 1 \tag{5}$$

must be fulfilled, where λ is the wavelength of the light in vacuum, and b is a mean dimension of the beam in the planar plane. Due to the second assumption, the incident beam as well as the secondary beams do not carry the spin AMs.

Let us represent the electric field vector of the incident beam $\mathbf{E}^{(i)}(\mathbf{r}; t)$, where t is the instant of time, as a superposition of the plane waves. By using this representation, the electric field vectors of the reflected and transmitted beams, $\mathbf{E}^{(\rho)}(\mathbf{r}; t)$ and $\mathbf{E}^{(\tau)}(\mathbf{r}; t)$ can be obtained after applying the Snell and the Fresnel laws to the particular plane waves of the incident beam. Further on we will operate simultaneously with the characteristics of the incident and secondary beams. In order to do that, it is convenient to introduce an auxiliary 2D vector $\boldsymbol{\kappa}$ whose components κ_x and κ_y are equal to the planar components of the wave vector $\mathbf{k}^{(i)}$ of the particular plane wave inside the incident beam. By using the auxiliary vector $\boldsymbol{\kappa}$, the vector $\mathbf{k}^{(i)}$ together with the vectors $\mathbf{k}^{(\rho)}$ and $\mathbf{k}^{(\tau)}$, which are connected with $\mathbf{k}^{(i)}$ by means of the Snell laws, can be represented in the unitary form:

$$\mathbf{k}^{(a)}(\boldsymbol{\kappa}) = \mathbf{k}_{\perp}^{(a)}(\boldsymbol{\kappa}) + k_z^{(a)}(\boldsymbol{\kappa})\hat{\mathbf{z}}^{(a)}. \tag{6}$$

The planar 2D vector $\mathbf{k}_{\perp}^{(a)}(\boldsymbol{\kappa})$ is expressed in terms of the auxiliary 2D vector $\boldsymbol{\kappa}$ as follows:

$$\mathbf{k}_{\perp}^{(a)}(\boldsymbol{\kappa}) = k_x^{(a)}(\boldsymbol{\kappa})\hat{\mathbf{x}}^{(a)} + \kappa_y\hat{\mathbf{y}}, \tag{7}$$

where

$$k_x^{(a)}(\boldsymbol{\kappa}) = \zeta^{(a)}\kappa_x. \tag{8}$$

Equation (7) with $k_x^{(a)}(\boldsymbol{\kappa})$ given by the linear relation (8) is exact for $a = i, \rho$, while it is approximate for $a = \tau$. Note that for the direct calculation of the TSCG of the transmitted beam such an approximation is insufficient (see [10]).

In order to simplify the forthcoming analysis, let us impose the following restrictions on the $z^{(i)}$-, $z^{(\rho)}$- and $z^{(\tau)}$- coordinates:

$$|z^{(a)}| \ll \frac{\pi b^2}{\lambda}. \tag{9}$$

Under such a restriction, i.e. in the near-field region, the dependence of $k_z^{(a)}$ on $\boldsymbol{\kappa}$ can be neglected; hence, $k_z^{(a)}(\boldsymbol{\kappa}) \simeq 2\pi\nu^{(a)}n^{(1)}/\lambda$.

By using expression (6) for the wave vector of the particular plane wave of the *a*th beam and the Fresnel laws, the electric field vectors of the incident and secondary beams can also be represented in the unitary form:

$$\mathbf{E}^{(a)}(\mathbf{r}; t) = \frac{C}{4\pi}\int \hat{\mathbf{e}}^{(a)}(\boldsymbol{\kappa})q^{(a)}(\boldsymbol{\kappa})F(\boldsymbol{\kappa})\exp\left[\mathrm{i}\left(\frac{2\pi}{\lambda}t - \mathbf{k}^{(a)}(\boldsymbol{\kappa})\cdot\mathbf{r}\right)\right]\mathrm{d}\boldsymbol{\kappa} + c.c. \tag{10}$$

In the right-hand side of equation (10) the following definitions are used. $\hat{\mathbf{e}}^{(a)}(\boldsymbol{\kappa})$ is the polarization vector of the particular plane wave inside the *a*th beam, which obeys the condition $\mathbf{k}^{(a)}(\boldsymbol{\kappa})\cdot\hat{\mathbf{e}}^{(a)}(\boldsymbol{\kappa}) = 0$; basing on this condition and using equations (6) and (7), one obtains that, in the first-order approximation,

$$\hat{\mathbf{e}}^{(a)}(\boldsymbol{\kappa}) = \hat{\mathbf{e}}^{(a)}(0) - \frac{\lambda k_{x,y}^{(a)}(\boldsymbol{\kappa})}{2\pi\nu^{(a)}n^{(1)}}\hat{\mathbf{z}}^{(a)}, \tag{11}$$

where $\hat{\mathbf{e}}^{(a)}(0) = \hat{\mathbf{x}}^{(a)}$ and $k^{(a)}_{x,y}(\boldsymbol{\kappa}) \equiv k^{(a)}_x(\boldsymbol{\kappa})$ in the case of p-polarization, and $\hat{\mathbf{e}}^{(a)}(0) = \hat{\mathbf{y}}$ and $k^{(a)}_{x,y}(\boldsymbol{\kappa}) \equiv \kappa_y$ in the case of s-polarization. The factors $q^{(i)}(\boldsymbol{\kappa})$, $q^{(\rho)}(\boldsymbol{\kappa})$ and $q^{(\tau)}(\boldsymbol{\kappa})$ are as follows: $q^{(i)}(\boldsymbol{\kappa}) = 1$, while $q^{(\rho)}(\boldsymbol{\kappa})$ and $q^{(\tau)}(\boldsymbol{\kappa})$ are the Fresnel field reflection and transmission coefficients of the s- or p-polarized incident plane wave with the wave vector $\mathbf{k}^{(i)}(\boldsymbol{\kappa})$ [23], the dependence of these quantities on the polarization is not explicitly pointed out. The function $F(\boldsymbol{\kappa})$ determines the field distribution inside the incident and, as a consequence, inside the reflected and transmitted beams. Let us assume that this function vanishes sufficiently quickly when $\kappa \to \infty$ (its width is of the order of π/b), and that it is normalized as follows:

$$\int |F(\boldsymbol{\kappa})|^2 \, \mathrm{d}\boldsymbol{\kappa} = 1. \tag{12}$$

If the incident beam carries the well-defined OAM,

$$F(\boldsymbol{\kappa}) = f(\kappa) \exp(-\mathrm{i}\sigma\varphi), \tag{13}$$

where σ is the azimuthal index ($\sigma = 0, \pm 1, \pm 2 \ldots$), and φ is the azimuth of the vector $\boldsymbol{\kappa}$ defined by the relation $\tan\varphi = \kappa_y/\kappa_x$, the function $f(\kappa)$ does not depend on φ. C is a normalization constant. The light velocity in vacuum is taken to be unity.

The magnetic field vector of the ath beam $\mathbf{H}^{(a)}(\mathbf{r}; t)$ is obtained by means of the Faraday law; once $\mathbf{E}^{(a)}(\mathbf{r}; t)$ is given by equation (10),

$$\mathbf{H}^{(a)}(\mathbf{r}; t) = \frac{\nu^{(a)} n^{(1)} C}{4\pi} \int \hat{\mathbf{u}}^{(a)}(\boldsymbol{\kappa}) q^{(a)}(\boldsymbol{\kappa}) F(\boldsymbol{\kappa}) \exp\left[\mathrm{i}\left(\frac{2\pi}{\lambda} t - \mathbf{k}^{(a)}(\boldsymbol{\kappa}) \cdot \mathbf{r}\right)\right] \mathrm{d}\boldsymbol{\kappa} + c.c., \tag{14}$$

where the unit vector

$$\hat{\mathbf{u}}^{(a)}(\boldsymbol{\kappa}) = \frac{\lambda}{2\pi \nu^{(a)} n^{(1)}} \mathbf{k}^{(a)}(\boldsymbol{\kappa}) \times \hat{\mathbf{e}}^{(a)}(\boldsymbol{\kappa}). \tag{15}$$

3.2. The electromagnetic energy and the LM of the ath beam

The electromagnetic energy density inside the ath beam is

$$w^{(a)}(\mathbf{r}; t) = \frac{1}{8\pi} [(\nu^{(a)} n^{(1)})^2 (\mathbf{E}^{(a)}(\mathbf{r}; t))^2 + (\mathbf{H}^{(a)}(\mathbf{r}; t))^2]. \tag{16}$$

Being cycle averaged, the electromagnetic energy density as well as the other characteristics of the ath beam do not depend on t. Further on, if the argument t is omitted, the respective characteristic of the beams will be assumed to be cycle averaged. The electromagnetic energy pul of the ath beam is given by

$$V^{(a)} = \int w^{(a)}(\mathbf{r}) \, \mathrm{d}\mathbf{r}^{(a)}_{\perp}, \tag{17}$$

where the 2D integration is performed over the cross-section of the ath beam. The limits of integration over y in the right-hand side of equation (17) are infinite. One of the limits of integration over $x^{(a)}$ is also infinite, while the other is finite. The latter is equal to the $x^{(a)}$-coordinate of the line of intersection of the respective cross-section with the interface, i.e. to $Z^{(a)}/\tan\theta^{(a)}$, where $Z^{(a)}$ is the $z^{(a)}$-coordinate of the chosen cross-sections of the ath beam. As a consequence, $V^{(a)}$ generally depends on $Z^{(a)}$ as well. Let us suppose that the cross-sections are situated far enough from the coordinate origin, i.e. the following conditions are fulfilled:

$$|Z^{(a)}| \gg b|\tan\theta^{(a)}|, \tag{18}$$

and, if $\theta^{(i)} < \pi/4$,

$$|Z^{(i,\rho)}| \gg b/\tan\theta^{(i)}. \tag{19}$$

Conditions (18) and (19) provide that the presence of the reflected field on the cross-section of the incited beam and vice versa can be neglected. Again, if condition (18) is fulfilled, the function $w^{(a)}(\mathbf{r}; t)$ is negligible at the line of intersection of the respective cross-section with the interface; in this case, the finite limit of integration over $x^{(a)}$ in equation (17) can be substituted by $-\infty$ or $+\infty$. Once such a substitution is made and expressions (10), (14) and (16) are used, the calculation of $V^{(a)}$ is straightforward. Let us neglect in equations (10) and (14) the dependence of the unit vectors $\hat{\mathbf{e}}^{(a)}(\boldsymbol{\kappa})$ and $\hat{\mathbf{u}}^{(a)}(\boldsymbol{\kappa})$ as well as of the field reflection and transmission coefficients $q^{(\rho)}(\boldsymbol{\kappa})$ and $q^{(\tau)}(\boldsymbol{\kappa})$ on $\boldsymbol{\kappa}$, and substitute these equations into the right-hand side of equation (16) and then equation (16) into the right-hand side of equation (17). In the result, let us first carry out the integration over $\mathbf{r}_\perp^{(a)}$, after that the following calculations will be straightforward. By performing these calculations one obtains that

$$V^{(i)} = \frac{(n^{(1)})^2|C|^2}{8\pi}, \tag{20}$$

while the relation between $V^{(a)}$ and $V^{(i)}$ is as follows:

$$V^{(a)} = \nu^{(a)} Q^{(a)} V^{(i)}, \tag{21}$$

where

$$Q^{(a)} = |\zeta^{(a)}|^{-1}\nu^{(a)}|q^{(a)}|^2. \tag{22a}$$

It is evident from the definitions of the quantities $\zeta^{(a)}$, $\nu^{(a)}$ and $q^{(a)}$ that $Q^{(i)} = 1$, while $Q^{(\rho)}$ and $Q^{(\tau)}$ are the reflectivity and transmissivity of the plane wave, which is incident at the angle $\theta^{(i)}$ and whose polarization vector is $\hat{\mathbf{e}}^{(i)}(0)$ [23]:

$$Q^{(\rho)} + Q^{(\tau)} = 1. \tag{22b}$$

Relation (21) is correct in the first-order approximation with respect to the parameter λ/b:

The density of the LM inside the ath beam is as follows:

$$\mathbf{g}^{(a)}(\mathbf{r}; t) = \frac{(\nu^{(a)}n^{(1)})^2}{4\pi}\mathbf{E}^{(a)}(\mathbf{r}; t) \times \mathbf{H}^{(a)}(\mathbf{r}; t). \tag{23}$$

The LM pul of the ath beam $\gamma^{(a)}$ is calculated similar to $V^{(a)}$; in the zero-order approximation with respect to λ/b,

$$\gamma^{(a)} = \int \mathbf{g}^{(a)}(\mathbf{r})\,\mathrm{d}\mathbf{r}_\perp^{(a)} = \nu^{(a)}n^{(1)}V^{(a)}\hat{\mathbf{z}}^{(a)}. \tag{24}$$

3.3. *The IOAM and the extrinsic OAM of the ath beam*

The density of the AM inside the ath beam, relative to the coordinate origin, is given by

$$\mathbf{m}^{(a)}(\mathbf{r}; t) = \mathbf{r} \times \mathbf{g}^{(a)}(\mathbf{r}; t). \tag{25}$$

The AM pul of the ath beam, relative to the same point is

$$\boldsymbol{\mu}^{(a)} = \int \mathbf{m}^{(a)}(\mathbf{r})\,\mathrm{d}\mathbf{r}_\perp^{(a)}. \tag{26}$$

Generally, the AM of the ath beam can be divided into the IAM and the extrinsic OAM (see [4, subsection 2.2]). Denoting the former and the latter by $\mathbf{j}^{(a)}$ and $\mathbf{l}^{(a)}$, respectively, we have

$$\boldsymbol{\mu}^{(a)} = \mathbf{j}^{(a)} + \mathbf{l}^{(a)}. \tag{27}$$

The vector $\mathbf{l}^{(a)}$ is defined as follows:

$$\mathbf{l}^{(a)} = \mathbf{R}_\perp^{(a)} \times \gamma^{(a)}, \tag{28}$$

where $\mathbf{R}_\perp^{(a)}$ is the 2D radius vector of the centre of gravity of the ath beam:

$$\mathbf{R}_\perp^{(a)} \equiv X^{(a)}\hat{\mathbf{x}}^{(a)} + Y^{(a)}\hat{\mathbf{y}} = \frac{1}{V^{(a)}}\int \mathbf{r}_\perp^{(a)} w^{(a)}(\mathbf{r})\,\mathrm{d}\mathbf{r}_\perp^{(a)}. \tag{29}$$

Note that, if relations (18) and (19) are fulfilled, the dependence of $X^{(a)}$ and $Y^{(a)}$ on $Z^{(a)}$ can be neglected. Hence, the vector $\mathbf{j}^{(a)}$ is given by

$$\mathbf{j}^{(a)} = \int \left(\mathbf{r}_\perp^{(a)} - \mathbf{R}_\perp^{(a)}\right) \times \mathbf{g}^{(a)}(\mathbf{r})\,\mathrm{d}\mathbf{r}_\perp^{(a)}. \tag{30}$$

Let us define the axis of the incident beam to be a line, which is the locus of its centre of gravity at a large distance from the coordinate origin. Under such a definition, $\mathbf{R}_\perp^{(i)} = 0$, hence, $\mathbf{l}^{(i)} = 0$; as for the vectors $\mathbf{R}^{(\rho)}$ and $\mathbf{R}_\perp^{(\tau)}$, they give the 2D shifts of the centres of gravity of the reflected and transmitted beams relative to respective geometric-optical axes.

Generally, the vector $\mathbf{R}_\perp^{(\rho,\tau)}$ contains two terms. One is independent of $Z^{(\rho,\tau)}$; it describes the linear 2D shift of the centre of gravity of the respective beam. The other one is proportional to $Z^{(\rho,\tau)}$ and to the beam's angular shift. In the near-field region (see equation (9)), the latter term can be neglected[1]. So, further on, we will suppose that $X^{(\rho,\tau)}$ and $Y^{(\rho,\tau)}$ are the linear shifts. $X^{(\rho)}$ corresponds to the Goos–Hänchen shift as concerns the beam's centre of gravity. $Y^{(\rho)}$ and $Y^{(\tau)}$ describe the TSCGs of the secondary beams [10–12].

The vectors $\gamma^{(\rho)}$ and $\gamma^{(\tau)}$ are approximately directed along the geometric-optical axes of the respective beams (see equation (24)). Hence, the shifts $X^{(a)}$ and $Y^{(a)}$ generate the extrinsic OAMs relative to the coordinate origin, which are parallel to the unit vectors $\hat{\mathbf{y}}$ and $\hat{\mathbf{x}}^{(a)}$, respectively.

As for the IAM pul of the ath beam, it is, in a general case, the sum of the spin AM and the IOAM. However, if the incident beam is s- or p-polarized, as is assumed in this paper, the spin AM of every beam is zero, hence, the vector $\mathbf{j}^{(a)}$ equals to the IOAM of the ath beam. This vector is approximately parallel to $\hat{\mathbf{z}}^{(a)}$, i.e., in the first-order approximation, $\mathbf{j}^{(a)} = j^{(a)}\hat{\mathbf{z}}^{(a)}$. Considering the transformation of the IOAM at reflection and transmission, it is convenient to operate with the dimensionless vectors $\boldsymbol{\Sigma}^{(i)}$, $\boldsymbol{\Sigma}^{(\rho)}$ and $\boldsymbol{\Sigma}^{(\tau)}$, which are defined as follows:

$$\boldsymbol{\Sigma}^{(a)} = \Sigma^{(a)}\hat{\mathbf{z}}^{(a)}, \tag{31a}$$

where

$$\Sigma^{(a)} = \frac{2\pi}{\lambda}\frac{j^{(a)}}{V^{(a)}}. \tag{31b}$$

Further on, the dimensionless quantity $\Sigma^{(a)}$ will be called the orbital number of the ath beam.

The calculations of the quantities $j^{(a)}$ and $\Sigma^{(a)}$ are performed in the following way. First, let us substitute equations (10) and (14) into the right-hand side of equation (23). We restrict ourselves with the nearest-order approximation. In this case, one can neglect, like by calculation of $V^{(a)}$ and $\gamma^{(a)}$, the dependence of the field reflection and transmission coefficients $q^{(\rho)}$ and $q^{(\tau)}$ on $\boldsymbol{\kappa}$. But, unlike by calculation of $V^{(a)}$ and $\gamma^{(a)}$, the dependence of the polarization vector of the ath beam on $\boldsymbol{\kappa}$ given by equation (11) must be taken into account. Making such an approximation and performing cycle averaging let us substitute the transformed equation (23) into the right-hand side of equation (30). The scalar product of the obtained vector $\mathbf{j}^{(a)}$ and of the unit vector $\hat{\mathbf{z}}^{(a)}$ will give an expression for the quantity $j^{(a)}$,

[1] In a subsequent analysis only the TSCGs of the secondary beams will be involved. In view of that, it should be noted that the spin-independent angular TSCGs of the reflected and transmitted beams have been predicted in [10, 24, 25]. Their results explicitly show that these shifts can be ignored if condition (9) is fulfilled.

which contains the 2D integration over $\mathbf{r}_\perp^{(a)}$ and two 2D integrations over the respective wave vectors. In this expression, let us use the relation

$$\mathbf{r}_\perp^{(a)} \exp\left(-\mathrm{i}\mathbf{k}_\perp^{(a)} \cdot \mathbf{r}\right) = \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\mathbf{k}_\perp^{(a)}} \exp\left(-\mathrm{i}\mathbf{k}_\perp^{(a)} \cdot \mathbf{r}\right)$$

and first carry out the integration over $\mathbf{r}_\perp^{(a)}$. After that, integration over one of the two 2D wave vectors will be straightforward; by performing this integration one will obtain the resulting formula for $j^{(a)}$. The substitution of this formula in the right-hand side of equation (31*b*) will give the expression for the orbital number of the *a*th beam, which can be represented as a sum of two terms:

$$\Sigma^{(a)} = \Sigma_{[xy]}^{(a)} + \Sigma_{[yx]}^{(a)}. \tag{32}$$

Every term $\Sigma_{[\alpha\beta]}^{(a)}$ ($\alpha = x, \beta = y$, or $\alpha = y, \beta = x$) is as follows:

$$\Sigma_{[\alpha\beta]}^{(a)} = \int F^*(\boldsymbol{\kappa})\dot{\Sigma}_{\alpha\beta}^{(a)} F(\boldsymbol{\kappa})\,\mathrm{d}\boldsymbol{\kappa}, \tag{33}$$

where $\dot{\Sigma}_{[xy]}^{(a)}$ and $\dot{\Sigma}_{[yx]}^{(a)}$ are the following operators:

$$\dot{\Sigma}_{[xy]}^{(a)} = \mathrm{i}k_x^{(a)}\frac{\mathrm{d}}{\mathrm{d}\kappa_y}, \qquad \dot{\Sigma}_{[yx]}^{(a)} = -\mathrm{i}\kappa_y\frac{\mathrm{d}}{\mathrm{d}k_x^{(a)}}. \tag{34}$$

Let us use the relation between $k_x^{(a)}$ and κ_x given by equation (8). By means of this relation, the operators $\dot{\Sigma}_{[xy]}^{(a)}$ and $\dot{\Sigma}_{[yx]}^{(a)}$ are written as follows:

$$\dot{\Sigma}_{[xy]}^{(a)} = \mathrm{i}\zeta^{(a)}\kappa_x\frac{\mathrm{d}}{\mathrm{d}\kappa_y}, \qquad \dot{\Sigma}_{[yx]}^{(a)} = -\mathrm{i}\frac{1}{\zeta^{(a)}}\kappa_y\frac{\mathrm{d}}{\mathrm{d}\kappa_x}. \tag{35}$$

After substituting equations (35) into the right-hand side of equation (33), we obtain the following relations between the respective parts of the orbital numbers of the secondary and the incident beams:

$$\Sigma_{[xy]}^{(\rho,\tau)} = \zeta^{(\rho,\tau)}\Sigma_{[xy]}^{(i)}, \qquad \Sigma_{[yx]}^{(\rho,\tau)} = \frac{1}{\zeta^{(\rho,\tau)}}\Sigma_{[yx]}^{(i)}. \tag{36}$$

Figures 2(*a*) and (*b*) illustrate relations between the vectors $\boldsymbol{\Sigma}_{[\alpha\beta]}^{(\rho)}$, $\boldsymbol{\Sigma}_{[\alpha\beta]}^{(\tau)}$ and $\boldsymbol{\Sigma}_{[\alpha\beta]}^{(i)}$ ($\boldsymbol{\Sigma}_{[\alpha\beta]}^{(a)} = \Sigma_{[\alpha\beta]}^{(a)}\hat{\mathbf{z}}^{(a)}$).

Relations (36) show that the transformation of the orbital number at reflection or transmission is determined by the factors $\zeta^{(\rho)}$ or $\zeta^{(\tau)}$ given by equation (2). Let us analyse these relations.

$\zeta^{(\rho)} = -1$; hence, at reflection, both parts of the orbital number of the incident beam trannsform equally. As a consequence, $\Sigma^{(\rho)} = -\Sigma^{(i)}$, i.e., in the zero-order approximation, the orbital numbers of the incident and reflected beams are of the same magnitudes, but of the opposite signs. The latter means that, in relation to the respective axes, the rotation motions of the energy inside these beams occur in the opposite directions.

Unlike $\zeta^{(\rho)}$, the factor $\zeta^{(\tau)}$ is positive. Thus, the direction of the rotational motion of the energy inside the transmitted beam in relation to its geometric-optical axis is the same as the direction of such a motion inside the incident beam relative to its axis. However, two parts of the orbital number of the incident beam are transformed differently at transmission. The latter effect can be explained in the following way. The magnitudes of the quantities $\Sigma_{[xy]}^{(a)}$ and $\Sigma_{[yx]}^{(a)}$ are estimated as follows:

$$\left|\Sigma_{[xy]}^{(a)}\right| \sim \overline{y}^{(a)}\overline{k}_x^{(a)}, \qquad \left|\Sigma_{[yx]}^{(a)}\right| \sim \overline{x}^{(a)}\overline{\kappa_y}^{(a)}, \tag{37}$$

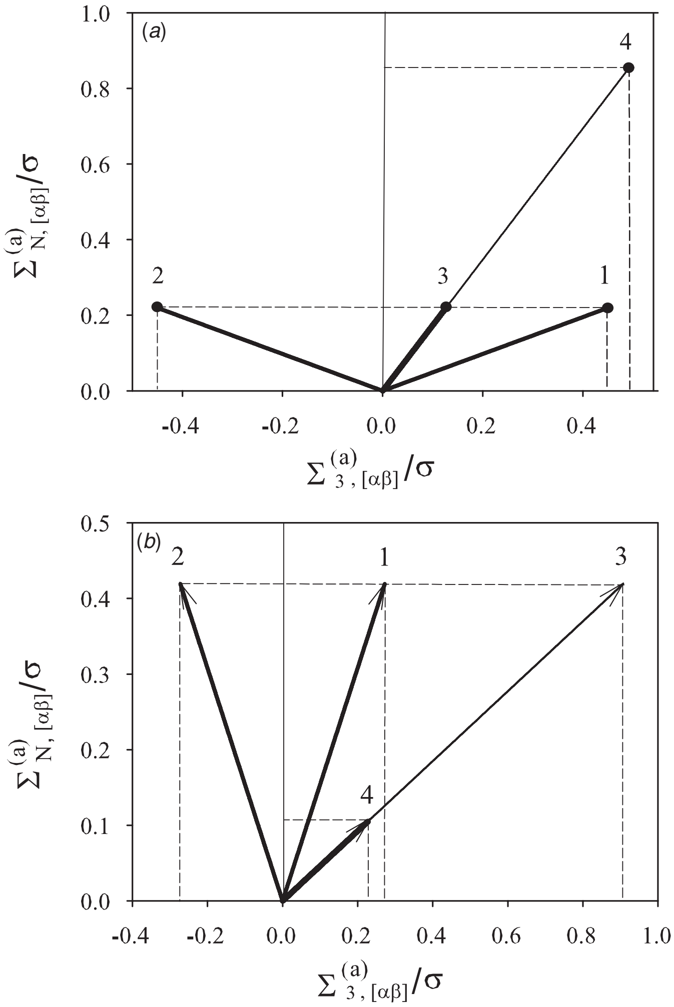

Figure 2. (*a*) Dimensionless vectors $\Sigma^{(i)}_{[\alpha\beta]}$, $\Sigma^{(\rho)}_{[\alpha\beta]}$ and $\Sigma^{(\tau)}_{[\alpha\beta]}$. Orthogonal components of these vectors are as follows: $\Sigma^{(a)}_{3,[\alpha\beta]} = (\hat{\mathbf{y}} \times \hat{\mathbf{N}}) \cdot \Sigma^{(a)}_{\alpha\beta}$ (the abscissa) and $\Sigma^{(a)}_{N,[\alpha\beta]} = \hat{\mathbf{N}} \cdot \Sigma^{(a)}_{\alpha\beta}$ (the ordinate); the measuring rules of the abscissa and the ordinate are different. The IOAM of the incident beam is assumed to be well defined: $n^{(2)}/n^{(1)} = 1.8$ and $\theta^{(i)} = 64^o$. 1–$\Sigma^{(i)}_{[xy]}$ or $\Sigma^{(i)}_{[yx]}$, 2–$\Sigma^{(\rho)}_{[xy]}$ or $\Sigma^{(\rho)}_{[yx]}$, 3–$\Sigma^{(\tau)}_{[xy]}$, 4–$\Sigma^{(\tau)}_{[yx]}$. (*b*) The same as in figure 2*a* but with $n^{(2)}/n^{(1)} = 0.6$, and $\theta^{(i)} = 33^o$.

where $\overline{k}^{(a)}_x$ and $\overline{\kappa}^{(a)}_y$ are the root-mean-square values of the wave vectors $k^{(a)}_x$ and κ_y inside the ath beam. $\overline{x}^{(a)}$ and $\overline{y}^{(a)}$ are the mean dimensions of the ath beam in $x^{(a)}$ and y directions; in the k space, $\overline{x}^{(a)}$ and $\overline{y}^{(a)}$ are given by the root-mean-square values of the operators $\mathrm{i}\,\mathrm{d}/\mathrm{d}k^{(a)}_x$ and $\mathrm{i}\,\mathrm{d}/\mathrm{d}\kappa_y$, respectively, inside the ath beam. However, the ratios of the y and x dimensions of the transmitted and the incident beam can be estimated on the basis of simple geometrical constructions, which show that, in the near-field region, $\overline{y}^{(\tau)} \simeq \overline{y}^{(i)}$, while $\overline{x}^{(\tau)} \simeq \overline{x}^{(i)}/\zeta^{(\tau)}$. The latter relation is explained in figure 3, while the former is evident. As for the ratios between the root-mean-square values of the wave vectors of the transmitted and incident beams, these can be estimated on the basis of the previous relations and of the uncertainty principle. In such

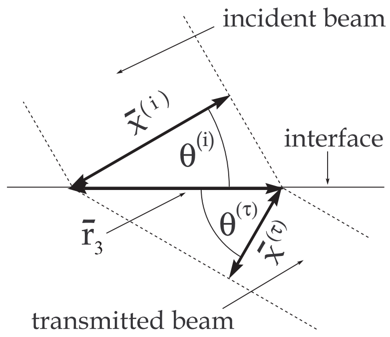

Figure 3. Geometrical interpretation of the relation $\overline{x}^{(\tau)} \simeq \overline{x}^{(i)}/\zeta^{(\tau)}$. Real beams are mentally replaced by beams with sharp borders; their diffusion is ignored. As can be seen from the figure, for such beams, the segment $\overline{r}_3 = \overline{x}^{(i)}/\cos\theta^{(i)}$ and $\overline{x}^{(\tau)} = \overline{r}_3\cos\theta^{(\tau)}$, thus, $\overline{x}^{(\tau)} = \overline{x}^{(i)}/\zeta^{(\tau)}$.

a way we obtain $\overline{k}_x^{(\tau)} \sim \overline{k}_x^{(i)}\zeta^{(\tau)}$, and $\overline{k}_y^{(\tau)} \sim \overline{k}_y^{(i)}$. By substituting the above-mentioned relations in equations (37), one obtains the approximate relations between the quantities $\left|\Sigma^{(\tau)}_{[\alpha\beta]}\right|$ and $\left|\Sigma^{(i)}_{[\alpha\beta]}\right|$, which are similar to relations (36).

Relations (36) are valid for an arbitrary incident beam, provided that it is paraxial and *s*- or *p*-polarized. If the IOAM of this beam is well defined, i.e. if the function $F(\boldsymbol{\kappa})$, which determines the field distribution inside the beam, is given by equation (13), the orbital number of the incident beam coincides with its azimuthal index, i.e.

$$\Sigma^{(i)} = 2\Sigma^{(i)}_{[xy]} = 2\Sigma^{(i)}_{[yx]} = \sigma. \tag{38}$$

In this case, $\Sigma^{(\rho)} = -\sigma$, and $\Sigma^{(\tau)} = 0.5(\cos\theta^{(i)}/\cos\theta^{(\tau)} + \cos\theta^{(\tau)}/\cos\theta^{(i)})\sigma$. The former relation indicates that, in the zero-order approximation, the IOAM of the reflected beam, such as of the incident one, is well defined. Thus, the field distribution inside this beam should be described by the function similar to that given by equation (13), but with the azimuthal index $-\sigma$; this fact is known (see, for instance, [26]). On the other hand, the IOAM of the transmitted beam is not well defined, and $\Sigma^{(\tau)}/\sigma > 1$.

4. Conservation laws at reflection and transmission of a light beam carrying the IOAM

4.1. Relation between the normal components of IOAMs and TSCGs of secondary beams

Let us analyse the results obtained in the previous section on the basis of the conservation laws, which take place in the process of reflection and transmission of the wave packets. As has been shown in [4] (see also the appendix of the present paper), the following quantities are the invariants of this process: the total electromagnetic energy, the components of the total LM parallel to the interface, and the normal to the interface component of the total AM. These quantities are defined as the integrals of the respective densities over the whole space, i.e. over the both the upper and the lower half-spaces (see the appendix). On the basis of the above-mentioned conservation laws, equation (20a) in [4] has been obtained, which establishes the relation between the TSCGs of the secondary packets and the changes of the normal components of IAMs, which take place at reflection and transmission. Note that, though this relation is valid in a general case, when the IAM of the incident beam is arbitrary (provided that this quantity is defined by equation (17) in [4]), the case, when the incident beam carries the IOAM, was not analysed in [4]. Here we will carry through such an analysis.

In order to apply the conservation laws, which have been established in [4] for the packets, to the characteristics of the beams, let us select a section of the incident beam of the length D, which is large in comparison with b but small in comparison with the Rayleigh range, and trace the motion of this section. Let us denote by $t^{(1)}$ an instant of time, at which the selection is made and assume that, at this instant, the selected section is situated far enough from the interface, so that the influence of the second medium on it can be neglected. Being separated from the beam, the selected section represents a packet of electromagnetic waves. The electric field vector of the respective packet $\tilde{\mathbf{E}}^{(a)}(\mathbf{r}; t)$ will be described by equation (10), if in its right-hand side the function F is supposed to depend on $\boldsymbol{\kappa}$ as well on λ (the width of this function should be of the order of λ^2/D) and the integration over λ is carried out[2]. After reflection or transmission of such a packet, the secondary packet will approximately represent the sections of the respective secondary beams. Let us fix another instant of time $t^{(2)}$, at which the secondary packets will be far enough from the interface so that the influence of the second medium on the reflected packet and of the first medium on the transmitted one can be neglected. It is convenient to introduce the unit notation $t^{(a)}$ for both instants of time $t^{(1)}$ and $t^{(2)}$ as follows: $t^{(a)} = t^{(1)}$ for $a = i$ and $t^{(a)} = t^{(2)}$ for $a = \rho, \tau$.

The distances of the incident and reflected packets from the coordinate origin at the instants $t^{(1)}$ and $t^{(2)}$, respectively, will be assumed to obey relations (18) and (19). Then, at these instants, the part of the total AM as well as the parts of the total electromagnetic energy and of the total LM, which originate from the interference of the incident and the reflected fields, can be neglected. Thus, at $t^{(1)}$ and $t^{(2)}$, the total AM (this quantity is given by equation (A7)) is as follows: $\mathbf{M}(t^{(1)}) = \mathbf{M}^{(i)}(t^{(1)})$ and $\mathbf{M}(t^{(2)}) = \mathbf{M}^{(\rho)}(t^{(2)}) + \mathbf{M}^{(\tau)}(t^{(2)})$. The conservation law of the normal component of $\mathbf{M}$ (see equation (A17)) leads to the relation

$$M_N^{(\rho)}(t^{(2)}) + M_N^{(\tau)}(t^{(2)}) = M_N^{(i)}(t^{(1)}). \tag{39}$$

One can easily be convinced that, in the zero-order approximation with respect to b/D, the relation between the vectors $\mathbf{M}^{(a)}(t^{(a)})$ and $\boldsymbol{\mu}^{(a)}$ is as follows:

$$\mathbf{M}^{(a)}(t^{(a)}) = \frac{D}{\nu^{(a)}}\boldsymbol{\mu}^{(a)}. \tag{40}$$

The factor $1/\nu^{(a)}$ in the right-hand side of equation (40) describes the change of the packet's length after its reflection or transmission. The lengths of the incident and reflected beams are approximately the same ($\nu^{(\rho)} = 1$). On the other hand, after transmission, the packet's length is changed by $1/\nu^{(\tau)} = n^{(1)}/n^{(2)}$.

A substitution of equation (40) into equation (39) yields the relation between the normal components of the vectors $\boldsymbol{\mu}^{(i)}$, $\boldsymbol{\mu}^{(\rho)}$ and $\boldsymbol{\mu}^{(\tau)}$. As can be seen from equation (27), the normal component of every vector $\boldsymbol{\mu}^{(a)}$ is the sum of the intrinsic $\left(j_N^{(a)}\right)$ and extrinsic $\left(l_N^{(a)}\right)$ parts.

Let us consider the latter one, which can easily be obtained from equations (24), (28) and the left relation (29). Due to the definition of the incident beam's axis, $Y^{(i)} = 0$ and $l_N^{(i)} = 0$. As for $l_N^{(\rho)}$ and $l_N^{(\tau)}$, they are generated by the TSCGs of the reflected and transmitted beams. (The extrinsic OAMs generated by the shifts $X^{(\rho)}$ and $X^{(\tau)}$, being perpendicular to $\hat{\mathbf{N}}$, are not involved in the balance of the normal component of the total AM.) For an arbitrary a,

$$l_N^{(a)} = -Y^{(a)}\gamma_3^{(a)}, \tag{41}$$

[2] We shall use the same letters for notations of field vectors and other characteristics of the ath beam and of the ath packet. However, in order to make the difference between the characteristics of the beam and of the packet, in the latter case the respective letter will be marked by the accent 'tilde'. Note that the characteristics of the packets, unlike the characteristics of the beams, even when cycle averaged, do depend on t.

where $\gamma_3^{(a)}$ is the component of the vector $\gamma^{(a)}$, which is parallel to the interface and to the plane of incidence: $\gamma_3^{(a)} = (\hat{\mathbf{y}} \times \hat{\mathbf{N}}) \cdot \gamma^{(a)}$. It is evident that

$$\gamma_3^{(a)} = \gamma^{(a)} \sin\theta^{(a)}. \tag{42}$$

As follows from equations (1), (21), (24) and (42), $\gamma_3^{(\rho)}$ and $\gamma_3^{(\tau)}$ are related to $\gamma_3^{(i)}$ by

$$\gamma_3^{(\rho,\tau)} = \nu^{(\rho,\tau)} Q^{(\rho,\tau)} \gamma_3^{(i)}. \tag{43}$$

Taking into account equations (22*b*) and (43) one obtains the following relation: $\gamma_3^{(\rho)} + \gamma_3^{(\tau)}/\nu^{(\tau)} = \gamma_3^{(i)}$, which represents, as a matter of fact, the conservation law (A19).

Let us substitute equation (27) into the right-hand side of equation (40) and use equations (41) and (43) in the obtained expression for the normal component of the vector $\boldsymbol{\mu}^{(a)}$; after that, let us substitute the expression for $M_N^{(\rho)}(t^{(2)})$, $M_N^{(\tau)}(t^{(2)})$ and $M_N^{(i)}(t^{(1)})$, into equation (39). Then, using relation (22*b*) and performing some transformations, we obtain the following relation:

$$Q^{(\rho)}(Y^{(\rho)} - h^{(\rho 1)}) = -Q^{(\tau)}(Y^{(\tau)} - h^{(\tau 1)}), \tag{44}$$

where

$$h^{(a1)} = \frac{j_N^{(a)}/\nu^{(a)} - Q^{(a)} j_N^{(i)}}{Q^{(a)}\gamma_3^{(i)}}. \tag{45}$$

In terms of the dimensionless vector $\boldsymbol{\Sigma}^{(a)}$, which is defined by equations (31*a*) and (31*b*), the quantity $h^{(a1)}$ can be written as follows:

$$h^{(a1)} = \frac{\left(\Sigma_N^{(a)} - \Sigma_N^{(i)}\right)\lambda}{2\pi n^{(1)} \sin\theta^{(i)}}. \tag{46}$$

4.2. The analysis

Let us discuss the general features of relation (44). Consider first the cases, when only one secondary beam is generated, i.e. when $Q^{(\rho)} = 1$ and $Q^{(\tau)} = 0$, or $Q^{(\tau)} = 1$ and $Q^{(\rho)} = 0$. The former situation takes place when $n^{(1)} > n^{(2)}$ and $\theta^{(i)}$ exceeds the critical angle for total reflection; the latter definitely takes place when the incident beam is *p*-polarized and $\theta^{(i)}$ is equal to the Brewster angle. In both cases equation (44) establishes a single-valued relation between the change of the IOAM at reflection or transmission and the TSCG of the respective secondary beam, in these cases, $Y^{(a)} = h^{(a1)}$. The last relation together with equation (45) are interpreted as follows. Let us consider the process of the total reflection or total transmission of the section of the incident beam of the unit length, which carries the IOAM equal to $\mathbf{j}^{(i)}$. After reflection or transmission, this section transforms into the section of the secondary beam of length $1/\nu^{(\rho,\tau)}$, which carries the IOAM equal to $\mathbf{j}^{(\rho,\tau)}/\nu^{(\rho,\tau)}$. So, after this process, the normal component of the IOAM is changed by $j_N^{(\rho,\tau)}/\nu^{(\rho,\tau)} - j_N^{(i)}$. As M_N must be conserved, the normal component of the extrinsic OAM of the respective section of the secondary beam given by equation (41) must compensate this change. This requirement together with the conservation law (A19), which is expressed, in the case under consideration, by relation (43) with $Q^{(\rho)} = 1$ or $Q^{(\tau)} = 1$, leads to the conclusion that the TSCG of the totally reflected or totally transmitted beam must be equal to the quantity $h^{(a1)}$ given by equation (45). It should be underlined that, if the IOAMs pul of the incident and of the single secondary beams are known, the TSCG of the totally reflected or totally transmitted beam can be obtained solely on the basis of the conservation laws without its direct calculation.

In the case when both the reflected and transmitted beams are generated, only the TSCG of the total secondary field, i.e. the quantity $Y = Q^{(\rho)}Y^{(\rho)} + Q^{(\tau)}Y^{(\tau)}$, can be obtained solely on the basis of the conservations laws. The single-valued definitions of the TSCGs of both secondary beams separately are impossible in such a way. However, in the two-secondary-beams regime, relation (44) imposes restrictions on the values of $Y^{(\rho)}$ and $Y^{(\tau)}$. Namely, in a general case the TSCG of every secondary beam must be the sum of two terms:

$$Y^{(a)} = h^{(a1)} + h^{(a2)}, \tag{47}$$

and the terms $h^{(\rho 2)}$ and $h^{(\tau 2)}$ must obey the relation

$$Q^{(\rho)} h^{(\rho 2)} + Q^{(\tau)} h^{(\tau 2)} = 0. \tag{48}$$

In the two-secondary-beams regime, the terms $h^{(\rho 1)}$ and $h^{(\tau 1)}$ have the same physical meaning as in the single-secondary-beam regime. Indeed, the intensity of the incident beam can mentally be divided into two parts: one, proportional to $Q^{(\rho)}$, which is to be reflected, and the other one, proportional to $Q^{(\tau)}$, which is to be transmitted. It is evident that the normal components of the extrinsic OAMs generated by the TSCGs $h^{(\rho 1)}$ and $h^{(\tau 1)}$ compensate the changes of the normal components of the IOAMs of the respective parts of the incident beam at their reflection and transmission. Thus, if $Y^{(a)}$ were equal to $h^{(a1)}$, the conservation of the normal component of the AM would take place in the processes of reflection or transmission separately. In view of that, the term $h^{(a1)}$ can be called the 'conservation TSCG' of the ath beam.

Calculations of the 'conservation TSCGs' of the secondary beams are easily performed on the basis of the results obtained in section 3. By substituting equation (31a) into the right-hand side of equation (46) and by using relations (36) one obtains that, in the case of an arbitrary $\theta^{(i)}$,

$$h^{(\rho 1)} = 0, \tag{49}$$

and

$$h^{(\tau 1)} = \Sigma^{(i)}_{[yx]} \tan\theta^{(i)} \left(1 - \left(\frac{n^{(1)}}{n^{(2)}}\right)^2\right) \frac{\lambda}{2\pi n^{(1)}}. \tag{50}$$

It can be seen from equation (50) that the 'conservation TSCG' of the transmitted beam is stipulated by the transformation of the vector $\mathbf{\Sigma}^{(i)}_{[yx]}$ but not of the vector $\mathbf{\Sigma}^{(i)}_{[xy]}$, because the normal components of the vectors $\mathbf{\Sigma}^{(i)}_{[xy]}$ and $\mathbf{\Sigma}^{(\tau)}_{[xy]}$ are equal. This fact is illustrated in figures 2(a) and (b). Bearing in mind both equations (49) and (50), the above conclusions may be generalized as follows: the transformation of only one part of the IOAM of the incident beam, namely of $\mathbf{j}^{(i)}_{[yx]}$, entails the 'conservation TSCG'. If the sign of $\Sigma^{(i)}_{[yx]}$ is fixed, the sign of $h^{(\tau 1)}$ depends on whether the second medium is more or less dense than the first one.

Let us consider the above-mentioned cases when the single secondary beam is generated. Equation (49) means that, because of the conservation of the normal component of the AM, the TSCG of the totally reflected beam must be equal to zero. On the other hand, in the case of the total transmission, i.e. when the incident beam is p-polarized and $\theta^{(i)}$ is equal to the Brewster angle, the TSCG of the transmitted beam is nonzero provided that $\Sigma^{(i)}_{[yx]}$ is nonzero; in this case, $Y^{(\tau)} = h^{(\tau 1)} = \lambda\Sigma^{(i)}_{[yx]}((n^{(2)}/n^{(1)})^2 - 1)/(2\pi n^{(2)})$. Approximately, the transmitted beam can also be considered as a single one when $|\Delta n|$ is small, where $\Delta n = n^{(2)} - n^{(1)}$; such a situation has been analysed by Bliokh [27]. Indeed, if $|\Delta n|/n^{(1)} \ll 1$, the reflectivity is small, namely, $Q^{(\rho)} \sim (\Delta n)^2$. Thus, in this case, due to relation (48), $h^{(\tau 2)} \sim (\Delta n)^2$. On the other hand, $h^{(\tau 1)} \sim \Delta n$, i.e. $|h^{(\tau 2)}|/|h^{(\tau 1)}| \ll 1$. Hence, if $|\Delta n|/n^{(1)} \ll 1$, the term $h^{(\tau 2)}$ can be neglected, and in this case, the expression for the TSCG of the transmitted beam looks like the following: $Y^{(\tau)} \simeq h^{(\tau 1)} = \lambda\Sigma^{(i)}_{[yx]} \tan\theta^{(i)} \Delta n/(n^{(1)})^2$.

Let us now move on to the situation when two secondary beams are generated. As the terms $h^{(\rho 2)}$ and $h^{(\tau 2)}$ cannot be separately defined on the basis of the conservation laws, let us turn to the direct calculations of the TSCGs. Such calculations, which have previously been carried out in [10], should be based on equation (29); their scheme is similar to the scheme of the calculations of the quantities $j^{(a)}$ and $\Sigma^{(a)}$. The difference is as follows: now the dependence of $q^{(\rho,\tau)}$ on κ_x and of $k_x^{(\tau)}$ on κ_y must be taken into account. On the other hand, the dependence of the polarization vector of the ath beam on $\boldsymbol{\kappa}$ can be ignored. The results of the calculations are as follows. $Y^{(\rho)}$ and $Y^{(\tau)}$ can be represented by equation (47). At that, $h^{(\rho 1)}$ and $h^{(\tau 1)}$ are given by equations (49) and (50), respectively, while

$$h^{(a2)} = -\frac{\lambda \Sigma^{(i)}_{[xy]}}{2\pi n^{(1)}} \frac{1}{Q^{(a)}(\vartheta)} \frac{\mathrm{d}Q^{(a)}(\vartheta)}{\mathrm{d}\vartheta}\bigg|_{\vartheta=\theta^{(i)}}, \tag{51}$$

where $Q^{(\rho)}(\vartheta)$ and $Q^{(\tau)}(\vartheta)$ denote the reflectivity and transmissivity of the s- or p-polarized plane wave, which is incident at an angle ϑ; in this denotation, $Q^{(a)}(\theta^{(i)}) \equiv Q^{(a)}$. As $Q^{(\rho)}(\vartheta) + Q^{(\tau)}(\vartheta) = 1$, it is evident that the terms $h^{(\rho 2)}$ and $h^{(\tau 2)}$ given by equation (51) satisfy condition (48).

It is interesting to compare the features and the physical meanings of the terms $h^{(a1)}$ and $h^{(a2)}$. The explanation of expression (51) for $h^{(\rho 2)}$, in the case when the incident beam carries the well-defined IOAM (i.e. when the function $F(\boldsymbol{\kappa})$ is described by equation (13)), can be found in [28] (see also [29, 30]). It is as follows. The distribution of the electromagnetic energy inside the incident beam carrying the well-defined IOAM is symmetrical relative to the plane of incidence. By calculating the vector $\boldsymbol{\Sigma}^{(\rho)}$ and, as a consequence, the 'conservation TSCG' of the reflected beam, we have neglected the dependence of the reflectivity on $\boldsymbol{\kappa}$. In this approximation, the distribution of the electromagnetic energy inside the reflected beam is symmetrical relative to the plane of incidence as well. If one takes into account the first-order effects, this distribution in the case of partial reflection becomes asymmetrical: the intensity increases at one side from the plane of incidence and decreases at the other one. This effect leads to the appearance of the TSCG $h^{(\rho 2)}$. The above-mentioned redistribution can be interpreted by means of the conceptions of a local angle of incidence and of local reflectivity [31] (see also [32, 33]). The key points of this interpretation are as follows: the azimuthal variation of the phase of the incident beam stipulates the variation of the local angle of incidence across the interface; this variation, in turn, stipulates the variation of the local reflectivity and, as a consequence, the modulation of the intensity profile of the reflected beam. The expression for $h^{(\tau 2)}$ can be interpreted in a similar way. Thus, the term $h^{(a2)}$ can be called the 'modulation TSCG'. In view of the above interpretation of the term $h^{(a2)}$, it is clear that this term should not depend on $\Sigma^{(i)}_{[yx]}$. Indeed, $\Sigma^{(i)}_{[yx]} \sim \kappa_y$ (see equations (33), (34)), however, in the first-order approximation, the transverse wave vector κ_y, unlike the vector κ_x, does not affect the angle of incidence of the particular plane wave and, as a consequence, the local angle of incidence.

Thus, the two parts of the IOAM of the incident beam, $\mathbf{j}^{(i)}_{[yx]}$ and $\mathbf{j}^{(i)}_{[xy]}$, stipulate different mechanisms of generation of the TSCG of the secondary beam: the 'conservation' and 'modulation' mechanisms, respectively; the 'conservation TSCG' of the reflected beam is zero.

The other difference between the 'conservation TSCG' and 'modulation TSCG' of the ath beam should also be pointed out. As can be seen from equations (49) and (50), $h^{(a1)}$, unlike $h^{(a2)}$, is the same in the cases of s- and p-polarizations, provided that the space structures of the incident beams and the angles of incidence are the same in both cases. The reason of this difference is as follows. $h^{(a2)}$ is controlled by the dependence of $Q^{(a)}(\vartheta)$ on ϑ (see equation (51)) while $h^{(\tau 2)}$ by the dependence of $k_x^{(\tau)}$ on κ_y. Note that the latter dependence can be neglected by the calculation of the IOAMs of the beams, as it is done in the present paper (see equation (8)).

4.3. Comparison of the IOAM-dependent and spin-dependent phenomena

In this paper, the spin-dependent TSCGs of the secondary beams were left out of consideration; it is because the incident beam was assumed to be *s*- or *p*-polarized. However, the general conclusions about the IOAM-dependent TSCGs of the secondary beams, which were based on the analysis of equations (44) and (45), are applied to the spin-dependent TSCGs as well. As a consequence, the spin-dependent TSCGs of the secondary beams, like the IOAM-dependent TSCGs, can be represented as sums of the 'conservation' and 'modulation' terms. At that, the spin-dependent 'modulation TSCGs' of the reflected and transmitted beams, like the IOAM-dependent 'modulation TSCGs', must obey relation (48). The direct calculations of the spin-dependent TSCGs, which have been performed in [34, 35], confirm the above conclusions.

However, there are significant differences between the IOAM-dependent and the spin-dependent TSCGs, which are of interest from the point of view of the conservation laws. The most important difference is as follows: when $h^{(\rho 1)}$ always equals zero, it is not the case for the spin-dependent 'conservation TSCG' of the reflected beam. The latter is not zero when the incident beam is elliptically polarized or, in the case of the total reflection, is linearly polarized with the polarization vector being oblique to the plane of incidence. Thus, in these cases, the spin-dependent TSCG of the totally reflected beam, which is usually called the Imbert–Fedorov shift [14, 15], is nonzero, and its value can be obtained solely on the basis of the conservation laws. This fact was recognized by de Beauregard [16], who called the spin-dependent TSCG of the totally reflected beam 'the translational inertial spin effect with photons'.

The other interesting difference between the spin-dependent TSCGs and the IOAM-dependent TSCGs takes place in the two-secondary-beams regime. Namely, in some cases, in particular in the important case when the incident beam is circularly polarized and its polarization does not depend upon y , the spin-dependent 'modulation TSCGs' of the reflected and transmitted beams are equal to zero. Thus, in these cases, the spin-dependent TSCG of every secondary beams looks as if the conservation law of the normal component of the total AM takes place for the processes of reflection and transmission separately. Recently, Onoda *et al* [17] have drawn attention to this fact; the conception of the 'separate conservation laws' has been discussed in [17–22]. For the OAM-dependent TSCGs, the above-mentioned situation does not take place.

5. Conclusions

We have considered the transformation of the OAM at reflection and transmission of a light beam at a plane interface of two isotropic transparent media. The incident beam was assumed to be *s*- or *p*-polarized; so neither this beam nor the secondary beams carry the spin IAMs. In this case, the AM pul of every beam consists of the IOAM and the extrinsic OAM. Relative to the coordinate origin, the latter is defined as the vector product of the radius vector of the respective beam's centre of gravity and the LM pul (see equation (28)). The IOAM and the extrinsic OAM are parallel and perpendicular, respectively, to the axis of the *a*th beam.

The IOAM pul of an arbitrary beam is characterized by the dimensionless vector $\mathbf{\Sigma}^{(a)}$ given by equation (31*a*). The value of this vector (the orbital number $\Sigma^{(a)}$) is given by equation (31*b*); if the IOAM of the incident beam is well defined, $\Sigma^{(i)}$ is equal to the azimuthal index σ. The orbital number of every beam is the sum of the two terms $\Sigma^{(a)}_{[xy]}$ and $\Sigma^{(a)}_{[yx]}$. We have obtained unitary expressions for the quantities $\Sigma^{(i)}$, $\Sigma^{(\rho)}$ and $\Sigma^{(\tau)}$. In the nearest-order approximation with respect to λ/b, the relations between the quantities $\Sigma^{(\rho,\tau)}_{[\alpha\beta]}$ and $\Sigma^{(i)}_{[\alpha\beta]}$ are given by equation (36), the features of these relations being as follows. The orbital numbers of the incident and reflected beams are of the same magnitudes but of opposite signs. The orbital numbers of the incident and transmitted beams are of the same signs. The relation between $\Sigma^{(\tau)}_{[xy]}$ and $\Sigma^{(i)}_{[xy]}$ differs from the relation between $\Sigma^{(\tau)}_{[yx]}$ and $\Sigma^{(i)}_{[yx]}$. A simple geometrical interpretation of the latter effect has been presented.

We have also analysed the transformation of the OAMs at reflection and transmission from the point of view of the conservation laws, which take place in these processes. The component of the total AM normal to the interface is conserved in the process of reflection and transmission. On the basis of this and other conservation laws, equation (44) has been derived. This equation together with equation (46) establishes a relation between the normal components of the dimensionless vectors $\mathbf{\Sigma}^{(a)}$ and the TSCGs of the secondary beams $Y^{(\rho,\tau)}$. In the case when one secondary beam is generated, this relation allows one to obtain the value of the TSCG of the respective beam without its direct calculation; in this case the TSCG is equal to the quantity $h^{(a1)}$ given by equation (49) or equation (50). In particular, it follows from equation (49) that the conservation law of the normal component of the AM requires that the IOAM-dependent TSCG of the totally reflected beam, unlike the spin-dependent one, must equal zero.

In the case when the reflected and transmitted beams are generated, the single-valued definitions of the TSCGs of both secondary beams are impossible solely on the basis of the conservation laws. However, in the two-secondary-beams regime, these laws impose the restrictions on the values of $Y^{(\rho)}$ and $Y^{(\tau)}$. Namely, due to relation (44), the TSCG of the ath beam must be the sum of two terms: the 'conservation TSCG' ($h^{(a1)}$) and the 'modulation TSCG' ($h^{(a2)}$). At that, the terms $h^{(\rho2)}$ and $h^{(\tau2)}$ must obey relation (48); owing to this relation, the 'modulation TSCGs' do not affect the TSCG of the total secondary field. As $h^{(\rho1)} = 0$, it is concluded based on the conservation laws that only the 'modulation' mechanism can be responsible for the generation of the TSCG of the reflected beam. On the other hand, both mechanisms are involved in the generation of the TSCG of the transmitted beam. Direct calculations of the TSCGs of the reflected and transmitted beams confirm these conclusions.

Acknowledgments

The author thanks M Onoda, S Murakami and K Y Bliokh for valuable correspondence. This work was supported by Estonian Science Foundation (grant no 6534).

Appendix. Derivation of the conservation laws

In order to derive the conservation laws, which take place in the process of reflection and transmission of the electromagnetic wave packet at a plane interface, it is convenient to employ the Cartesian coordinate system connected with the interface. Let us define the coordinates in this system as follows: $r_1 = -\hat{\mathbf{N}}\cdot\mathbf{r}$, $r_2 \equiv y$ and $r_3 = (\hat{\mathbf{y}}\times\hat{\mathbf{N}})\cdot\mathbf{r}$.

Let us denote by $\tilde{\mathbf{E}}(\mathbf{r}; t)$ and $\tilde{\mathbf{H}}(\mathbf{r}; t)$ the electric and magnetic field vectors of the total electromagnetic field at an arbitrary point $\mathbf{r}$ and at an instant t:

$$\tilde{\mathbf{E}}(\mathbf{r}; t) = \tilde{\mathbf{E}}^{(i)}(\mathbf{r}; t) + \tilde{\mathbf{E}}^{(\rho)}(\mathbf{r}; t), \qquad \tilde{\mathbf{H}}(\mathbf{r}; t) = \tilde{\mathbf{H}}^{(i)}(\mathbf{r}; t) + \tilde{\mathbf{H}}^{(\rho)}(\mathbf{r}; t), \tag{A.1}$$

if $r_1 > 0$, and

$$\tilde{\mathbf{E}}(\mathbf{r};t) = \tilde{\mathbf{E}}^{(\tau)}(\mathbf{r};t), \qquad \tilde{\mathbf{H}}(\mathbf{r};t) = \tilde{\mathbf{H}}^{(\tau)}(\mathbf{r};t), \tag{A.2}$$

if $r_1 < 0$.

The electromagnetic energy density of the total field is

$$\tilde{w}(\mathbf{r};t) = \frac{1}{8\pi}[\epsilon(r_1)(\tilde{\mathbf{E}}(\mathbf{r};t))^2 + (\tilde{\mathbf{H}}(\mathbf{r};t))^2], \tag{A.3}$$

the density of the total LM is

$$\tilde{\mathbf{g}}(\mathbf{r};t) = \frac{\epsilon(r_1)}{4\pi}\tilde{\mathbf{E}}(\mathbf{r};t) \times \tilde{\mathbf{H}}(\mathbf{r};t), \tag{A.4}$$

and the density of the total AM is

$$\tilde{\mathbf{m}}(\mathbf{r};t) = \mathbf{r} \times \tilde{\mathbf{g}}(\mathbf{r};t), \tag{A.5}$$

where $\varepsilon(r_1)$ is the dielectric constant at an arbitrary point: $\varepsilon(r_1) = (n^{(1)})^2$, if $r_1 > 0$ and $\varepsilon(r_1) = (n^{(2)})^2$, if $r_1 < 0$.

At an arbitrary instant t, the total LM and the total AM are as follows:

$$\tilde{\mathbf{G}}(t) = \int \tilde{\mathbf{g}}(\mathbf{r};t)\,\mathrm{d}\mathbf{r}, \tag{A.6}$$

and

$$\tilde{\mathbf{M}}(t) = \int \tilde{\mathbf{m}}(\mathbf{r};t)\,\mathrm{d}\mathbf{r}. \tag{A.7}$$

The 3D integrations in equations (A6) and (A7) are performed over the whole space.

Conservation laws of the respective components of the LM and the AM are obtained on the basis of the local conservation law of the LM, which looks as follows [4, 8, 9]:

$$\frac{\partial \tilde{g}_j(\mathbf{r};t)}{\partial t} = \sum_{k=1}^{3} \frac{\partial \tilde{\eta}_{jk}(\mathbf{r};t)}{\partial r_k}, \tag{A.8}$$

where $j, k = 1, 2, 3$, and $\tilde{\eta}_{jk}(\mathbf{r};t)$ is the Maxwellian stress tensor,

$$\tilde{\eta}_{jk}(\mathbf{r};t) = \frac{1}{4\pi}(\varepsilon(r_1)\tilde{E}_j(\mathbf{r};t)\tilde{E}_k(\mathbf{r};t) + \tilde{H}_j(\mathbf{r};t)\tilde{H}_k(\mathbf{r};t) - \delta_{jk}\tilde{w}(\mathbf{r};t)), \tag{A.9}$$

where δ_{jk} is the Kronecker symbol.

Derivation of the conservation law of the normal to the interface component of the total AM is carried out in the following way. Let us take expression (A5) for the normal component of the density of the total AM, i.e. $\tilde{m}_1(t) \equiv -\tilde{m}_N(t)$, and act on the left-hand and right-hand sides of this expression with the operator $\partial/\partial t$. In the right-hand side of the obtained equation, let us use the expressions for $\partial \tilde{g}_2(\mathbf{r};t)/\partial t$ and $\partial \tilde{g}_3(\mathbf{r};t)/\partial t$ given by equation (A8) and take into account that $\tilde{\eta}_{23}(\mathbf{r};t) = \tilde{\eta}_{32}(\mathbf{r};t)$, what is evident from equation (A9). Then we get the following local conservation law for the normal component of the total AM:

$$\frac{\partial \tilde{m}_1(\mathbf{r};t)}{\partial t} = \sum_{k=1}^{3} \frac{\partial \tilde{\Pi}_{1k}(\mathbf{r};t)}{\partial r_k}, \tag{A.10}$$

where

$$\tilde{\Pi}_{1k}(\mathbf{r};t) = r_2\tilde{\eta}_{3k}(\mathbf{r};t) - r_3\tilde{\eta}_{2k}(\mathbf{r};t). \tag{A.11}$$

Let us integrate both sides of equation (A10) over the whole space. After this operation, the partial derivative in the left-hand side $\partial/\partial t$ can be replaced by the full one $\mathrm{d}/\mathrm{d}t$. In the right-hand side of the obtained equation, 3D integrals transforms, by means of the Gauss theorem, into 2D integrals over two half-spheres of the infinite radii and over the both sides of the interface. The integrals over half-spheres of the infinite radii equal zero; hence,

$$\frac{\mathrm{d}\tilde{M}_N(t)}{\mathrm{d}t} = \int_{-\infty}^{\infty} \mathrm{d}r_2 \int_{-\infty}^{\infty} \mathrm{d}r_3 \big(\tilde{\Pi}_{11}(|r_1|, r_2, r_3; t) - \tilde{\Pi}_{11}(-|r_1|, r_2, r_3; t)\big)|_{|r_1|\to 0}. \tag{A.12}$$

(In the left-hand side of equation (A12) the evident relation $\tilde{M}_1(t) \equiv -\tilde{M}_N(t)$ was used).

Let us now take into account the border conditions on the interface [23], which can be written as follows:

$$[\tilde{E}_{2,3}(|r_1|, r_2, r_3; t) = \tilde{E}_{2,3}(-|r_1|, r_2, r_3; t)]|_{|r_1|\to 0}, \tag{A.13}$$

$$[\varepsilon(|r_1|)\tilde{E}_1(|r_1|, r_2, r_3; t) = \varepsilon(-|r_1|)\tilde{E}_1(-|r_1|, r_2, r_3; t)]|_{|r_1|\to 0}, \tag{A.14}$$

and

$$[\tilde{\mathbf{H}}(|r_1|, r_2, r_3; t) = \tilde{\mathbf{H}}(-|r_1|, r_2, r_3; t)]|_{|r_1|\to 0}. \tag{A.15}$$

By the use of relations (A13)–(A15), one can be convinced that $\tilde{\eta}_{21}(\mathbf{r}; t)$ and $\tilde{\eta}_{31}(\mathbf{r}; t)$ are continuous on the interface, as a consequence, $\tilde{\Pi}_{11}(\mathbf{r}; t)$ is continuous as well, i.e.

$$[\tilde{\Pi}_{11}(|r_1|, r_2, r_3; t) = \tilde{\Pi}_{11}(-|r_1|, r_2, r_3; t)]|_{|r_1|\to 0}. \tag{A.16}$$

Thus, the right-hand side of equation (A12) equals zero; hence,

$$\frac{\mathrm{d}\tilde{M}_N(t)}{\mathrm{d}t} = 0. \tag{A.17}$$

Derivation of the conservation laws of the components of the total LM, which are parallel to the interface, is carried out in the similar way; these laws look as follows:

$$\frac{\mathrm{d}\tilde{G}_2(t)}{\mathrm{d}t} = 0, \tag{A.18}$$

and

$$\frac{\mathrm{d}\tilde{G}_3(t)}{\mathrm{d}t} = 0. \tag{A.19}$$

It should be mentioned that the conservation laws (A17)–(A19) have been derived without specification of the incident wave packet. Hence, they are valid for the beam carrying the IOAM as well as for the beam carrying the spin IAM.

References

[1] Allen L *et al* 1992 *Phys. Rev.* A **45** 8185
[2] Allen L 2002 *J. Opt. B: Quantum Semiclass. Opt.* **4** S1
[3] Allen L, Barnet S M and Padgett M J (ed) 2003 *Optical Angular Momentum* (Bristol: Institute of Physics Publishing)
[4] Fedoseyev V G 1988 *J. Phys. A: Math. Gen.* **21** 2045
[5] Fedoseyev V G 1985 *Preprint* F-30, Department of Physics and Astronomy Academy of Science of Estonian SSR
[6] Player M A 1987 *J. Phys. A: Math. Gen.* **20** 3667
[7] Hugrass W N 1990 *J. Mod. Opt.* **37** 339
[8] Moller C 1972 *The Theory of Relativity* (Oxford: Clarendon)
[9] Ginsburg V L 1973 *Usp. Fiz. Nauk* **110** 309
Ginsburg V L 1973 *Sov. Phys.—Usp.* **16** 434
[10] Fedoseyev V G 2001 *Opt. Commun.* **193** 9
[11] Bekshaev A and Popov A 2002 *Proc. SPIE* **4607** 90
[12] Bekshaev A Y and Popov A Y 2003 *Ukr. J. Phys. Opt.* **3** 249

[13] Dasgupta R and Gupta P K 2006 *Opt. Commun.* **257** 91
[14] Fedorov F I 1955 *Dokl. Akad. Nauk* **105** 465
[15] Imbert C 1972 *Phys. Rev.* D **5** 787
[16] de Beauregard O C 1965 *Phys. Rev.* **139** B1443
[17] Onoda M, Murakami S and Nagaosa N 2004 *Phys. Rev. Lett.* **93** 083901
[18] Bliokh K Y and Bliokh Y P 2006 *Phys. Rev. Lett.* **96** 073903
[19] Onoda M, Murakami S and Nagaosa N 2006 *Phys. Rev.* E **74** 066610
[20] Duval C, Horvath Z and Horvathy P A 2006 *Phys. Rev.* D **74** 021701
[21] Bliokh K Y and Bliokh Y P 2007 *Phys. Rev.* E **75** 06660
[22] Hosten O and Kwiat P 2008 *Science* **319** 787
[23] Born M and Wolf E 1965 *Principles of Optics* 3rd edn (Oxford: Pergamon)
[24] Alda J 2000 *Opt. Commun.* **182** 1
[25] Alda J and Rico-Garcia J M 2002 *Opt. Commun.* **213** 229
[26] Padgett M J and Lesso J P 1999 *J. Mod. Opt.* **46** 175
[27] Bliokh K Y 2006 *Phys. Rev. Lett.* **97** 043901
[28] Fedoseyev V G 2001 *Proc. SPIE* **4403** 90
[29] Okuda H and Sasada H 2006 *Opt. Express* **14** 8392
[30] Okuda H and Sasada H 2008 *J. Opt. Soc. Am.* A **25** 881
[31] Fedoseyev V G 2004 *Proc. SPIE* **5481** 154
[32] Marienko I G *et al* 2002 *Opt. Commun.* **213** 1
[33] Fedoseyev V G 2002 *Abstracts of the Int. 'Seminar Day on Difraction-2002'* (St Petersburg, Russia) p 28
[34] Fedoseyev V G 1991 *Opt. Spectrosk.* **71** 829
Fedoseyev V G 1991 *Opt. Spectrosc.* **71** 483
[35] Fedoseyev V G 1991 *Opt. Spectrosk.* **71** 992
Fedoseyev V G 1991 *Opt. Spectrosc.* **71** 570